On the origin of the difference between time and space

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We suggest that the difference between time and space is due to spontaneous symmetry breaking. In a theory with spinors the signature of the metric is related to the signature of the Lorentz-group. We discuss a higher symmetry that contains pseudo-orthogonal groups with arbitrary signature as subgroups. The fundamental asymmetry between time and space arises then as a property of the ground state rather than being put into the formulation of the theory a priori. We show how the complex structure of quantum field theory as well as gravitational field equations arise from spinor gravity - a fundamental spinor theory without a metric.

In special and general relativity time and space are treated in a unified framework. Nevertheless, a basic asymmetry between these two concepts persists, related to the signature of the metric. It is at the root of much of the complexity of physics and the universe. The quantum field equations with a Euclidean signature often admit as solutions only a single ground state or a few (sometimes degenerate) states. In contrast, the Minkowski signature allows for many complex solutions with causal time evolution. Within the presently existing attempts to find a unified theory of all interactions, based on quantum field or superstring theories, this time-space asymmetry is assumed a priori in the form of a given signature.

In this letter we pursue the perhaps radical idea that the difference between time and space arises as a consequence of the "dynamics" of the theory rather than being put in by hand. More precisely, we will discuss a model where the "classical" or "microscopic" action does not make any difference between time and space. The time-space asymmetry is generated only as a property of the ground state and can be associated to spontaneous symmetry breaking.

In a model with only bosonic fields the idea of associating the Minkowski signature to a ground state property may seem straightforward. One could discuss a quantum theory for a symmetric second rank tensor (metric) field which can take arbitrary positive or negative values for its elements. The expectation value of the metric defines then its signature and the ground state could indeed correspond to a Minkowski signature for an appropriate model. The situation changes drastically, however, in the presence of fermionic fields. The coupling of fermions to the gravitational degrees of freedom involves the vielbein e^m_μ. Both the spinors and the vielbein transform nontrivially under the "Lorentz group". In turn, the connection between the vielbein and the metric, $g_{\mu\nu} = e^m_\mu e^n_\nu \eta_{mn}$, involves the invariant tensor η_{mn} of this Lorentz group. The signature of the metric is then uniquely related to the signature of η_{mn} and therefore fixed once the Lorentz group is specified.

In consequence, a fermionic model for the spontaneous generation of the time-space asymmetry should not have a "bias" for a particular signature of the Lorentz group. We will discuss actions that are invariant under the complex orthogonal group $SO(d, \mathbb{C})$. This non-compact group admits as subgroups all pseudo-orthogonal groups $SO(s, d-s)$ with arbitrary signature s. However, the ground state may spontaneously break this symmetry, being invariant only with respect to pseudo-orthogonal transformations with a given s. Within our particular 16-dimensional model we will even give a reason why the ground state may favor the "physical signature" $s = 1$ as compared to the Euclidean signature $s = 0$.

Our approach is based on spinor gravity - a theory involving only fundamental fermions and no fundamental metric [1]-[5]. Nevertheless, the action is invariant under general coordinate tranformations (diffeomorphisms). The vielbein and the metric appear as composite objects involving two or four fermions. As a specific model we consider a 16-dimensional theory where the fermions are represented by 256 Grassmann variables $\psi_\alpha(x^\mu),\ \alpha = 1 \dots 256,\ \mu = 0 \dots 15,\ \{\psi_\alpha(x), \psi_\beta(y)\} = 0$. Below, we will construct an action $S[\psi]$ as an element of a real Grassmann algebra, i.e. as a "sum" (or integral) of polynomials in ψ with real coefficients. In a first step we will require that the action is invariant under the group $SO(128, \mathbb{C})$ generated by the infinitesimal transformations

$$\delta\psi = \begin{pmatrix} \rho, & -\tau \\ \tau, & \rho \end{pmatrix} \psi \tag{1}$$

with 128×128 blocks of real antisymmetric matrices $\rho = -\rho^T, \tau = -\tau^T$. This is a huge non-compact group with $128 \cdot 127$ real generators.

The group $SO(128, \mathbb{C})$ has as subgroups all pseudoorthogonal groups $SO(s, 16-s)$ for arbitrary signature s. Indeed, we may first embed $SO(16, \mathbb{C})$ into $SO(128, \mathbb{C})$ by restricting $(m, n = 0 \dots 15, \epsilon_{mn} = -\epsilon_{nm}, \bar{\epsilon}_{mn} = -\bar{\epsilon}_{nm})$

$$\rho = -\frac{1}{2}\epsilon_{mn}\hat{\Sigma}^{mn}\ ,\ \tau = \frac{1}{2}\bar{\epsilon}_{mn}\hat{\Sigma}^{mn}. \tag{2}$$

Here $\hat{\Sigma}^{mn} = -\hat{\Sigma}^{nm}$ are the generators of $SO(16)$ in the 128-component Majorana-Weyl representation [6]. They are represented by real antisymmetric 128×128-matrices. (For $SO(d)$ the real $2^{\frac{d}{2}-1}$-dimensional spinor representation exists only if $d = 8\ mod\ 8$.) The 240 real generators

of $SO(16,\mathbb{C})$ can be taken as

$$\begin{aligned}\Sigma_E^{mn} &= \hat{\Sigma}^{mn}\,1\ ,\ B^{mn}=-\hat{\Sigma}^{mn}I,\\ I &= \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix},\ I^2=-1.\end{aligned} \tag{3}$$

They obey the commutation relations

$$\begin{aligned}[][\Sigma_E^{mn},\Sigma_E^{pq}] &= f^{mnpqst}\Sigma_E^{st},\\ [B^{mn},B^{pq}] &= -f^{mnpqst}\Sigma_E^{st},\\ [\Sigma_E^{mn},B^{pq}] &= f^{mnpqst}B^{st}\end{aligned} \tag{4}$$

with f^{mnpqst} the usual structure constants of $SO(16)$ (with (mn) etc. considered as double-index). The pseudo-orthogonal group $SO(s,d-s)$ with signature s obtains by taking $\epsilon_{mn}^{(s)}=\epsilon_{mn}$ for $m\geq s, n\geq s$, $\epsilon_{mn}^{(s)}=-\epsilon_{mn}$ for $m<s, n<s$ and $\epsilon_{mn}^{(s)}=\bar{\epsilon}_{mn}$ otherwise, i.e. if one index is smaller and the other larger or equal s.

At this point everything is formulated within a real Grassmann algebra and no particular signature is singled out - the different s just denote different subgroups of the symmetry group of the action, $SO(128,\mathbb{C})$. Within our spinor theory the ground state may be characterized by non-vanishing expectation values of (bosonic) spinor bilinears or, more generally, by composites with even powers of Grassmann variables. Let us consider the bilinears

$$\tilde{E}_\mu^0=\psi_\alpha\partial_\mu\psi_\alpha\ ,\ \tilde{E}_\mu^k=\psi_\alpha(\hat{a}^kI)_{\alpha\beta}\partial_\mu\psi_\beta \tag{5}$$

where $\hat{a}^k$ denote 15 real antisymmetric 128×128-matrices obeying the anticommutation relation

$$\left\{\hat{a}^k,\hat{a}^l\right\}=-2\delta^{kl}\ ,\ k,l=1\ldots15. \tag{6}$$

Such matrices $\hat{a}^k$ exist since $\hat{\gamma}^k=i\hat{a}^k$ spans the Clifford algebra which admits for $d=15$ a 128-component spinor representation with purely imaginary and antisymmetric $\hat{\gamma}^k$ [6]. We can use $\hat{a}^k$ for the construction of the $SO(16)$ generators

$$\hat{\Sigma}^{kl}=\frac{1}{4}[\hat{a}^k,\hat{a}^l]\ ,\ \hat{\Sigma}^{0k}=-\frac{1}{2}\hat{a}^k \tag{7}$$

with $\hat{\Sigma}^{01}\hat{\Sigma}^{23}\ldots\hat{\Sigma}^{14,15}=1/256$. In particular, it is obvious that $\tilde{E}_\mu^k$ transforms under global $SO(15)$ as a 15-dimensional vector whereas $\tilde{E}_\mu^0$ is a singlet.

Actually, the bilinear $\tilde{E}_\mu^m$ transforms under global $SO(1,15)$ as a 16-dimensional Lorentz-vector. This can be seen by writing

$$\begin{aligned}\tilde{E}_\mu^m &= \bar{\psi}\beta^m\partial_\mu\psi\ ,\ \bar{\psi}=\psi^T\beta^0\ ,\\ \beta^0 &= \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\beta^k=\beta_0\hat{a}^kI\end{aligned} \tag{8}$$

where

$$\begin{aligned}\{\beta^m,\beta^n\} &= -2\eta_M^{mn}\ ,\ [\beta^m,\beta^n]=\frac{1}{4}\Sigma_M^{mn}\\ \eta_M^{mn} &= diag(-1,1\ldots1),\\ \Sigma_M^{0k} &= \frac{1}{2}\hat{a}^kI=B^{0k}\ ,\ \Sigma_M^{kl}=\Sigma_E^{kl}.\end{aligned} \tag{9}$$

We observe that for an $SO(1,15)$ Lorentz-transformation

$$\delta_{\mathcal{L}}\psi=-\frac{1}{2}\epsilon_{mn}^M\Sigma_M^{mn}\psi\ ,\ \delta_{\mathcal{L}}\bar{\psi}=\frac{1}{2}\epsilon_{mn}^M\bar{\psi}\Sigma_M^{mn} \tag{10}$$

one obtains indeed

$$\delta_{\mathcal{L}}\tilde{E}_\mu^m=-\tilde{E}_\mu^n(\epsilon^M)_n{}^{\ m}\ ,\ (\epsilon^M)_n{}^{\ m}=\epsilon_{np}^M\eta_M^{pm}. \tag{11}$$

We emphasize that our model does not allow the construction of the vector representation of $SO(16)$ from spinor bilinears. Only 15 matrices $\hat{a}^k$ obeying eq. (6) exist. On a group theoretical level the vector representation is contained in the product of two inequivalent irreducible spinor representations of $SO(s,16-s)$. From eq. (1) we see that for $SO(128)$ (as represented by ρ) and all its subgroups including $SO(16)$ the fermion field ψ is reducible into two *identical* real 128-dimensional representations. In contrast, for $s=1$ the 256-component Majorana spinor is equivalent to a 128-dimensional complex Weyl-spinor which is not equivalent to its complex conjugate. The product of two spinors therefore contains a vector for $SO(1,15)$ but not for $SO(16)$.

This simple algebraic property could be the root[12] of the observed asymmetry between time and space! Indeed, consider an expectation value of the form

$$E_\mu^m(x)=\langle\tilde{E}_\mu^m(x)\rangle=\delta_\mu^m. \tag{12}$$

It is left invariant if the global Lorentz transformation (10), (11) is accompanied by a suitable general coordinate transformation

$$\delta_\xi E_\mu^m(x)=-\xi^\nu(x)\partial_\nu E_\mu^m(x)-\partial_\mu\xi^\nu(x)E_\nu^m(x). \tag{13}$$

For a global Lorentz transformation of the coordinates the ground state (12) remains indeed invariant

$$\xi^\nu=(\epsilon^M)^\nu{}_{\ \mu}x^\mu\ ,\ (\delta_\xi+\delta_{\mathcal{L}})E_\mu^m=0. \tag{14}$$

The transformation property (11) carries over to the global vielbein $E_\mu^m(x)=\langle\tilde{E}_\mu^m(x)\rangle$ and we can construct a metric which is invariant under $\delta_{\mathcal{L}}$ by using the $SO(1,15)$ invariant tensor $\eta_{M,mn}$

$$g_{\mu\nu}(x)=E_\mu^m(x)E_\nu^n(x)\eta_{Mmn}. \tag{15}$$

A ground state $g_{\mu\nu}=\eta_{M\mu\nu}$ is then invariant under the coordinate transformation (14).

Let us next construct an action S for spinor gravity according to the following principles: i)S is an element of a real Grassmann algebra, (ii) S is invariant under general coordinate transformations (diffeomorphisms) $\delta_\xi\psi=-\xi^\nu\partial_\nu\psi$, (iii) S is invariant under global $SO(128,\mathbb{C})$ transformations (1), (iv) S is local. Under these conditions we will find that S can only be a sum of six invariants which involve either 144 or 274 powers of ψ. We also find that S is actually invariant under *local* $SO(128,\mathbb{C})$ transformations if two of the six couplings vanish.

For this construction it is useful to exploit the complex structure which is compatible with the transformation (1). Complex conjugation can be associated to an involution $\psi_{128+\hat{\alpha}} \to -\psi_{128+\hat{\alpha}}, \hat{\alpha} = 1 \dots 128$, i.e.

$$\psi \to K\psi \ , \ K = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ , \ K^2 = 1. \tag{16}$$

We can map K-odd quantities onto purely imaginary variables and introduce an 128- component *complex* Grassmann variable

$$\varphi_{\hat{\alpha}} = \psi_{\hat{\alpha}} + i\psi_{128+\hat{\alpha}} \ , \ \varphi^*_{\hat{\alpha}} = \psi_{\hat{\alpha}} - i\psi_{128+\hat{\alpha}}. \tag{17}$$

The transformation (1) can now be written as a complex matrix multiplication

$$\delta\varphi_{\hat{\alpha}} = \sigma_{\hat{\alpha}\hat{\beta}}\varphi_{\hat{\beta}} \ , \ \sigma = \rho + i\tau \tag{18}$$

which is compatible with the complex structure, $\delta\varphi^* = \sigma^*\varphi^*$. The σ are arbitrary complex antisymmetric 128×128 matrices, explaining the name $SO(128, \mathbb{C})$. Invariants under global $SO(128, \mathbb{C})$ transformations involve an appropriate number of $\varphi_{\hat{\alpha}}$ contracted with the two invariant tensors $\delta^{\hat{\alpha}\hat{\beta}}$ or $\epsilon^{\hat{\alpha}1\dots\hat{\alpha}_{128}}$ - in contrast, contractions of mixed terms involving φ and φ^* are not invariant. Diffeomorphism symmetry is realized if precisely 16 derivatives of φ are contracted with $\epsilon^{\mu_1\cdots\mu_{16}}$ and we find the general form of the invariant action

$$\begin{aligned} S &= \alpha \int d^d x W[\varphi] R(\varphi, \varphi^*) + c.c., \\ W[\varphi] &= \frac{1}{16!}\epsilon^{\mu_1\dots\mu_{16}}\partial_{\mu_1}\varphi_{\hat{\alpha}_1}\dots\partial_{\mu_{16}}\varphi_{\hat{\alpha}_{16}}L^{\hat{\alpha}_1\dots\hat{\alpha}_{16}}. \end{aligned} \tag{19}$$

Here L is the totally symmetric invariant tensor of rank 16

$$L^{\hat{\alpha}_1\dots\hat{\alpha}_{16}} = sym\left\{\delta^{\hat{\alpha}_1\hat{\alpha}_2}\delta^{\hat{\alpha}_3\hat{\alpha}_4}\dots\delta^{\hat{\alpha}_{15}\hat{\alpha}_{16}}\right\} \tag{20}$$

and *sym* denotes total symmetrization in all indices. The remaining part $R(\varphi, \varphi^*)$ is a local polynomial not involving derivatives. Due to the anticommuting properties the contractions can only involve the ϵ-tensor

$$\begin{aligned} R(\varphi, \varphi^*) &= T(\varphi) + \tau T(\varphi^*) + \kappa T(\varphi)T(\varphi^*), \\ T(\varphi) &= \frac{1}{128!}\epsilon^{\hat{\beta}_1\dots\hat{\beta}_{128}}\varphi_{\hat{\beta}_1}\dots\varphi_{\hat{\beta}_{128}}. \end{aligned} \tag{21}$$

(For $R = 1$ the action (19) becomes a total derivative.) We note $\left(\left(\alpha W[\varphi]T(\varphi)\right)^* = \alpha^* W[\varphi^*]T(\varphi^*)\right.$ etc.) that by use of eq. (17) the action (19) can be explicitely written as an element of a real Grassmann algebra for the variables ψ_α.

The action is characterized by three dimensionless complex (or six real) coupling constants α, τ and κ. For $\tau = 0$ it is invariant under *local* $SO(128, \mathbb{C})$ transformations. Due to the identity $\varphi_{\hat{\alpha}}(x)\varphi_{\hat{\alpha}}(x) = 0$ (no summation over $\hat{\alpha}$ here, also note $\hat{\varphi}_\alpha\hat{\varphi}^*_\alpha = 2i\psi_{128+\hat{\alpha}}\psi_{\hat{\alpha}} \neq 0$) at most 128 powers of φ can occur at a given location x such that the inhomogeneous contribution from $\delta_{\mathcal{L}}(\partial_\mu\varphi)$ does not contribute to $\delta_{\mathcal{L}}S$. (For details see [5].) Local Lorentz symmetry, if free of anomalies, has important consequences for the spectrum of gravitational excitations. Indeed, a model with only global Lorentz symmetry leads to additional massless gravitational degrees of freedom [4, 5]. We concentrate here on the case of local $SO(128, \mathbb{C})$ symmetry with $\tau = 0$. Imposing an additional discrete symmetry can further reduce the number of allowed couplings. In particular, the transformations $\varphi_1 \leftrightarrow \varphi_2$ or $\varphi_1 \to -\varphi_1$ are not part of $SO(128, \mathbb{C})$ and map $W \leftrightarrow W, T \leftrightarrow -T$. The reflection of one coordinate $x^0 \to -x^0$ results in $W \leftrightarrow -W, T \leftrightarrow T$. Requiring invariance under the combined transformation restricts $\kappa = 0$. Finally, a reflection symmetry $\varphi \leftrightarrow \varphi^*$ (16) maps $W \leftrightarrow W^*, T \leftrightarrow T^*$ and would leave only one real coupling α. This can be scaled to an arbitrary value (e.g. $\alpha = 1$) by an appropriate scaling of ψ.

The existence of only a small number of allowed dimensionless couplings strongly suggests that the theory may be renormalizable! After a proper regularization of the model (i.e. specification of a well defined functional measure) the couplings may evolve in dependence on some appropriate "renormalization scale" μ. The existence of a (partial) infrared fixed point would further enhance the predictive power of the model even for the case of several couplings. If the partial fixed point has precisely one marginal direction the corresponding dimensionless coupling would behave similar to the gauge coupling in QCD and could generate a characteristic mass scale by dimensional transmutation. All dimensionless quantities (like ratios of mass scales) would become, in principle, predictable, similar to the case of a single real coupling α. We also note that more invariants (with less powers of ψ) become possible if the symmetry of the action is only a subgroup of $SO(128, \mathbb{C})$, like $SO(16, \mathbb{C})$, which would be sufficient for our purpose.

The action $S(19)$ can be viewed as an element of a complex Grassmann algebra. Complex conjugation is defined by $\varphi \to \varphi^*$, accompanied by a complex conjugation of all coefficients. (This is the meaning of c.c. in eq. (19) such that $S^* = S$.) Hermitean conjugation involves an additional transposition of all Grassmann variables. We can extend this to a more general form of the action and note $S^\dagger = -S^*$ if S is a sum of elements with $n = 2 \ mod \ 4$ powers of φ or φ^*, whereas $S^\dagger = S^*$ for $n = 4 \ mod \ 4$. The bilinear ($k \neq 0$)

$$\begin{aligned} \tilde{E}^0_\mu &= \frac{1}{2}(\varphi^\dagger\partial_\mu\varphi + \varphi^T\partial_\mu\varphi^*), \\ \tilde{E}^k_\mu &= \frac{i}{2}(\varphi^\dagger\hat{a}^k\partial_\mu\varphi - \varphi^T\hat{a}^k\partial_\mu\varphi^*) \end{aligned} \tag{22}$$

is antihermitean and real. The Dirac matrices $\gamma^m = i\beta^m$ are purely imaginary and obey $\{\gamma^m, \gamma^n\} = 2\eta^{mn}_M$.

The generating functionals for the vielbein and its

(connected) correlation functions are defined as

$$Z[J] = \int \mathcal{D}\psi \exp\left\{-S_E + \int dx\, J^\mu_m(x)\tilde{E}^m_\mu(x)\right\} \quad (23)$$

and $W[J] = \ln Z[J]$. (Here S_E is related to the Minkowski action $S_M = iS_E$ and one could require $S_M^\dagger = S_M$.) The vielbein (12) obtains as

$$E^m_\mu(x) = \frac{\delta W}{\delta J^\mu_m(x)} \quad (24)$$

and obeys the gravitational field equations which follow from the variation of the effective action Γ, i.e.

$$\Gamma[E^m_\mu] \;=\; -W[J] + \int dx\, J^\mu_m(x)E^m_\mu(x), \quad (25)$$

$$\frac{\delta\Gamma}{\delta E^m_\mu(x)} \;=\; 0. \quad (26)$$

We emphasize that eq. (26) constitutes the quantum gravitational equation. All quantum fluctuations of spinor gravity are already incorporated into the computation of the effective action Γ - which is, of course, the difficult and challenging task. The above construction can be extended to other bosonic and fermionic sources and fields such that Γ contains all information about the correlation functions of interest.

If S and the regularization of the functional measure $\int \mathcal{D}\psi$ in eq. (23) preserve the symmetries of diffeomorphisms and local Lorentz transformations the effective action Γ is invariant. Expanding in derivatives of the vielbein, Γ is dictated by the symmetries to take the familiar form

$$\Gamma = \int d^{16}x \det(E^m_\mu)(c_1 + c_2 R + \ldots), \quad (27)$$

with R the curvature scalar formed from the metric (15) and $(\det(E^m_\mu))^2 = -\det(g_{\mu\nu})$. Dots denote higher invariant powers of the curvature tensor and its covariant derivatives. This is a familiar 16-dimensional action for the metric with Einstein and cosmological constant term.

For a realistic model the static "ground state" solution of the gravitational field equation (26,27) should preserve the Poincaré transformations acting on x^0 and three "spatial" coordinates (x^1, x^2, x^3), while the remaining 12 other coordinates could be associated to an internal space with characteristic length scale of the order of the Planck mass. The isometries of the internal geometry would show up as gauge symmetries in the dimensionally reduced effective four-dimensional theory [7]. For example, if nine coordinates form a subspace S^9 and two more a subspace S^2 the four-dimensional gauge symmetry would consist of an $SO(10)$ grand unification with an $SO(3)$ generation group. The chirality index [8] counting the number of massless chiral fermions could be nonvanishing, especially if the geometry is "wharped" [9] and internal space "non-compact" [10].

In conclusion, we have proposed a model where the asymmetry between time and space arises as a ground state property rather than being assumed a priori. In our treatment, there is no difference between "Euclidean time" and "Minkowski time" - both are described by a common real time coordinate x^0. The "physical signature" arises as a consequence of the expectation value of the bilinear $\tilde{E}^m_\mu$. Analytic continuation from the physical space-time to Euclidean space-time is achieved by analytically continuing E^0_μ from its real physical value to an imaginary value, $E^0_\mu \to iE^0_\mu$.

We have argued that our model of spinor gravity is an interesting candidate for a renormalizable theory of quantum gravity. In contrast to earlier approaches we have realized *local* Lorentz symmetry for a well defined action involving only spinors, i.e. for S an element of the Grassmann algebra which must be polynomial in the spinor fields. Our 16-dimensional model also has the potential to provide a unified picture for the observed gauge and gravitational interactions. Further steps towards a regularized functional measure and a computation of the effective action are needed before the search for the ground state can be attacked realiably. Along the lines of [11] our approach may also shed more light on the emergence of a unitary time evolution.

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[12] Ground states with $SO(16)$ symmetry are possible if bilinears with a different tensor struture condense. Also

$SO(15)$ or subgroups of it like $SO(4)$ can be left invariant by suitable expectation values of $\tilde{E}_{\mu}^{m}$.